FUNCTIONS

Q SET - 1

01) if $f(x) = ax + 5$ and $f(1) = 8$, find a — ans : 3

02) if $f(x) = lx - 4$ and $f(2) = 10$, find l — ans : 7

03) if $f(x) = 3x + a$ and $f(1) = 7$, find a and f(4) — ans : 4 , 16

04) if $f(x) = ax^2 + bx + 1$, $f(1) = 15$ and $f(-1) = 3$. Find a and b — ans : 8 , 6

05) if $f(x) = ax^2 + bx + 2$, $f(1) = 3$ and $f(4) = 42$. Find a and b — ans : 3 , –2

Q SET - 2

01) if $f(x) = x^2 - 3x + 5$. Solve the equation $f(x) = f(x + 1)$ — ans : 1

02) if $f(x) = x^2 + 5x - 7$. Solve the equation $f(x) = f(x - 1)$ — ans : –2

03) if $f(x) = x^2 - 3x + 4$. Find the value of x satisfying $f(x) = f(2x + 1)$ — ans : 2/3,–1

04) if $f(x) = x^2 - 4x + 11$. Solve the equation $f(x) = f(3x - 1)$ — ans : 1/2 , 5/4

Q SET - 3

01) $f(x) = 2x$; $g(x) = 4x + 1$. Find fog & gof — ans : $8x + 2$, $8x + 1$

02) $f(x) = 3x - 1$; $g(x) = x^2 + 1$. Find fog & gof — ans : $3x^2 + 2$, $9x^2 - 6x + 2$

03) $f(x) = x - 5$; $g(x) = x^2 - 1$. Find fog & gof — ans : $x^2 - 6$, $x^2 - 10x + 24$

04) $f(x) = 8x^3$; $g(x) = \sqrt[3]{x}$. Find fog & gof — ans : $8x$, $2x$

05) $f(x) = 256x^4$; $g(x) = \sqrt{x}$. Find fog & gof — ans : $256\,x^2$, $16x^2$

Q SET - 4

01) if $f(x) = \dfrac{2x + 3}{3x - 2}$; $x \neq 2/3$; Show that $f(f(x)) = x$ ..... fof is an identity function

02) if $f(x) = \dfrac{2x + 1}{5x - 2}$; $x \neq 2/5$; Show that $(fof)(x) = x$

03) if $f(x) = \dfrac{3x+4}{5x-7}$ and $g(x) = \dfrac{7x+4}{5x-3}$, Show that : $fog(x) = gof(x) = x$

04) if $f(x) = \dfrac{3x+2}{4x-1}$ and $g(x) = \dfrac{x+2}{4x-3}$, Show that : $fog(x) = gof(x) = x$

05) if $f(x) = \dfrac{x+3}{4x-5}$ and $g(x) = \dfrac{3+5x}{4x-1}$, Show that : $fog(x) = gof(x) = x$

Q SET - 5 : Find the range of the function

01) $f(x) = 5x - 3$; $-5 \leq x \leq 1$ ans : Range of f is $[-28, 2]$

02) $f(x) = 2x + 6$; $-1 \leq x \leq 5$ ans : Range of f is $[4, 16]$

03) $f(x) = 3 - 4x$; $-4 \leq x \leq 2$ ans : Range of f is $[-5, 19]$

04) $f(x) = -2 - 7x$; $-2 \leq x \leq 4$ ans : Range of f is $[-30, 12]$

05) $f(x) = 3x^2 + 5$; $-3 \leq x \leq 4$ ans : Range of f is $[5, 53]$

06) $f(x) = 2x^2 - 4$; $1 \leq x \leq 4$ ans : Range of f is $[-2, 28]$

07) $f(x) = 1 - 4x^2$; $-2 \leq x \leq 2$ ans : Range of f is $[-15, 1]$

08) $f(x) = 7 - 2x^2$; $2 \leq x \leq 4$ ans : Range of f is $[-25, -1]$

09) $f(x) = 2 - 5x^2$; $-1 \leq x \leq 3$ ans : Range of f is $[-43, 2]$

10) $f(x) = 9 - 2x^2$; $-5 \leq x \leq 3$ ans : Range of f is $[-41, 9]$

11) $f(x) = x^2 + 4x + 5$, $x \in R$ ans : Range of f is $[1, \infty)$

12) $f(x) = x^2 - 8x + 10$, $x \in R$ ans : Range of f is $[-6, \infty)$

13) $f(x) = x^2 - 6x + 11$, $x \in R$ ans : Range of f is $[2, \infty)$

Q SET - 6 : Find the inverse of the function

01) $f(x) = 2x + 5$ ans : $f^{-1}(x) = \frac{1}{2}(x - 5)$

02) $f(x) = \frac{2x}{3} + 5$ ans : $f^{-1}(x) = \frac{3}{2}(x - 5)$

03) $f(x) = \frac{3x}{4} - 7$ ans : $f^{-1}(x) = \frac{4}{3}(x + 7)$

04) $f(x) = 3x - 4$. Find $f^{-1}(x)$. Also find $f^{-1}(9)$ and $f^{-1}(-2)$

SOLUTION TO Q SET - 1

01) if $f(x) = ax + 5$ and $f(1) = 8$, find a

SOLUTION :

$f(x) = ax + 5$

$f(1) = 8$

$a(1) + 5 = 8$

$a = 3$

02) if $f(x) = lx - 4$ and $f(2) = 10$, find l

SOLUTION :

$f(x) = lx - 4$

$f(2) = 10$

$l(2) - 4 = 10$

$2l = 14$ $\quad \therefore\ l = 7$

03) if $f(x) = 3x + a$ and $f(1) = 7$, find a and f(4)

SOLUTION :

$f(x) = 3x + a$

$f(1) = 7$

$3(1) + a = 7$

$a = 4$ $\quad \therefore\ f(x) = 3x + 4$

$f(4) = 3(4) + 4 = 16$

04) if $f(x) = ax^2 + bx + 1$, $f(1) = 15$ and $f(-1) = 3$. Find a and b

SOLUTION :

$f(x) = ax^2 + bx + 1$

$f(1) = 15$	$f(-1) = 3$
$a(1)^2 + b(1) + 1 = 15$	$a(-1)^2 + b(-1) + 1 = 3$
$a + b = 14$	$a - b = 2$
…….(1)	……… (2)

Solving (1) & (2) : $a = 8$ & $b = 6$

05) if $f(x) = ax^2 + bx + 2$, $f(1) = 3$ and $f(4) = 42$. Find a and b

SOLUTION :

$f(x) = ax^2 + bx + 2$

$f(1) = 3$	$f(4) = 42$
$a(1)^2 + b(1) + 2 = 3$	$16a + 4b + 2 = 42$
$a + b = 1$	$16a + 4b = 40$
…….(1)	$4a + b = 10$ ……… (2)

Solving (1) & (2) : $a = 3$ & $b = -2$

SOLUTION TO Q SET - 2

01) if $f(x) = x^2 - 3x + 5$. Solve the equation $f(x) = f(x+1)$

SOLUTION :

$$f(x) = f(x+1)$$
$$x^2 - 3x + \cancel{5} = (x+1)^2 - 3(x+1) + \cancel{5}$$
$$\cancel{x^2} - \cancel{3x} = \cancel{x^2} + 2x + 1 - \cancel{3x} - 3$$
$$0 = 2x - 2$$
$$2x = 2 \qquad \therefore\ x = 1$$

02) if $f(x) = x^2 + 5x - 7$. Solve the equation $f(x) = f(x-1)$

SOLUTION :

$$f(x) = f(x-1)$$
$$x^2 + 5x - \cancel{7} = (x-1)^2 + 5(x-1) - \cancel{7}$$
$$\cancel{x^2} + \cancel{5x} = \cancel{x^2} - 2x + 1 + \cancel{5x} - 5$$
$$0 = -2x - 4$$
$$2x = -4 \qquad \therefore\ x = -2$$

04) if $f(x) = x^2 - 3x + 4$. Find the value of x satisfying $f(x) = f(2x+1)$

SOLUTION :

$$f(x) = f(2x+1)$$
$$x^2 - 3x + \cancel{4} = (2x+1)^2 - 3(2x+1) + \cancel{4}$$
$$x^2 - 3x = 4x^2 + 4x + 1 - 6x - 3$$
$$x^2 - 3x = 4x^2 - 2x - 2$$
$$3x^2 + x - 2 = 0$$
$$3x^2 + 3x - 2x - 2 = 0$$
$$3x(x+1) - 2(x+1) = 0$$
$$(3x-2)(x+1) = 0 \qquad \therefore \qquad x = 2/3 \ , -1$$

05) if $f(x) = x^2 - 4x + 11$. Solve the equation $f(x) = f(3x-1)$

SOLUTION :

$$f(x) = f(3x-1)$$
$$x^2 - 4x + 11 = (3x-1)^2 - 4(3x-1) + 11$$
$$x^2 - 4x = 9x^2 - 6x + 1 - 12x + 4$$
$$x^2 - 4x = 9x^2 - 18x + 5$$
$$8x^2 - 14x + 5 = 0$$
$$8x^2 - 4x - 10x + 5 = 0$$
$$4x(2x-1) - 5(2x-1) = 0$$
$$(2x-1)(4x-5) = 0 \qquad \therefore\ x = 1/2 \ , 5/4$$

SOLUTION TO Q SET - 3

01) $f(x) = 2x$; $g(x) = 4x + 1$. Find fog & gof

SOLUTION :

$$\begin{aligned} fog(x) &= f(g(x)) \\ &= 2g(x) \\ &= 2(4x + 1) \\ &= 8x + 2 \end{aligned}$$

$$\begin{aligned} gof(x) &= g(f(x)) \\ &= 4f(x) + 1 \\ &= 4(2x) + 1 \\ &= 8x + 1 \end{aligned}$$

02) $f(x) = 3x - 1$; $g(x) = x^2 + 1$. Find fog & gof

SOLUTION :

$$\begin{aligned} fog(x) &= f(g(x)) \\ &= 3(g(x) - 1 \\ &= 3(x^2 + 1) - 1 \\ &= 3x^2 + 3 - 1 \\ &= 3x^2 + 2 \end{aligned}$$

$$\begin{aligned} gof(x) &= g(f(x)) \\ &= (f(x))^2 + 1 \\ &= (3x - 1)^2 + 1 \\ &= 9x^2 - 6x + 1 + 1 \\ &= 9x^2 - 6x + 2 \end{aligned}$$

03) $f(x) = x - 5$; $g(x) = x^2 - 1$. Find fog & gof

SOLUTION :

$$\begin{aligned} fog(x) &= f(g(x)) \\ &= (g(x) - 5 \\ &= x^2 - 1 - 5 \\ &= x^2 - 6 \\ &= 3x^2 + 2 \end{aligned}$$

$$\begin{aligned} gof(x) &= g(f(x)) \\ &= (f(x))^2 - 1 \\ &= (x - 5)^2 - 1 \\ &= x^2 - 10x + 25 - 1 \\ &= x^2 - 10x + 24 \end{aligned}$$

04) $f(x) = 8x^3$; $g(x) = \sqrt[3]{x}$. Find fog & gof

SOLUTION :

$$\begin{aligned} fog(x) &= f(g(x)) \\ &= 8(g(x))^3 \\ &= 8\left(\sqrt[3]{x}\right)^3 \\ &= 8x \end{aligned}$$

$$\begin{aligned} gof(x) &= g(f(x)) \\ &= \sqrt[3]{f(x)} \\ &= \sqrt[3]{8x^3} \\ &= 2x \end{aligned}$$

05) $f(x) = 256x^4$; $g(x) = \sqrt{x}$. Find fog & gof

SOLUTION :

$$fog(x) = f(g(x)) = 256(g(x))^4 = 256\left(\sqrt{x}\right)^4 = 256\,x^2$$

$$gof(x) = g(f(x)) = \sqrt{f(x)} = \sqrt{256x^4} = 16x^2$$

SOLUTION TO Q SET - 4

01) if $f(x) = \dfrac{2x+3}{3x-2}$; $x \neq 2/3$; Show that $f(f(x)) = x$ ..... fof is an identity function

SOLUTION :

$$f(f(x)) = \frac{2f(x)+3}{3f(x)-2}$$

$$= \frac{2\left(\dfrac{2x+3}{3x-2}\right) + 3}{3\left(\dfrac{2x+3}{3x-2}\right) - 2}$$

$$= \frac{\dfrac{4x+6+9x-6}{3x-2}}{\dfrac{6x+9-6x+4}{3x-2}}$$

$$= \frac{13x}{13} = x = \text{RHS}$$

02) if $f(x) = \dfrac{2x+1}{5x-2}$; $x \neq 2/5$; Show that $(fof)(x) = x$

SOLUTION :

$$f(f(x)) = \frac{2f(x)+1}{5f(x)-2}$$

$$= \frac{2\left(\dfrac{2x+1}{5x-2}\right) + 1}{5\left(\dfrac{2x+1}{5x-2}\right) - 2}$$

$$= \frac{\dfrac{4x+2+5x-2}{5x-2}}{\dfrac{10x+5-10x+4}{5x-2}}$$

$$= \frac{9x}{9} = x = \text{RHS}$$

04) if $f(x) = \dfrac{3x + 4}{5x - 7}$ and $g(x) = \dfrac{7x + 4}{5x - 3}$, Show that : $fog(x) = gof(x) = x$

SOLUTION :

$$fog(x) = f(g(x))$$
$$= \frac{3g(x) + 4}{5g(x) - 7}$$
$$= \frac{3\left(\frac{7x + 4}{5x - 3}\right) + 4}{5\left(\frac{7x + 4}{5x - 3}\right) - 7}$$
$$= \frac{\frac{21x + 12 + 20x - 12}{5x - 3}}{\frac{35x + 20 - 35x + 21}{5x - 3}}$$
$$= \frac{41x}{41}$$
$$= x$$

$$gof(x) = g(f(x))$$
$$= \frac{7f(x) + 4}{5f(x) - 3}$$
$$= \frac{7\left(\frac{3x + 4}{5x - 7}\right) + 4}{5\left(\frac{3x + 4}{5x - 7}\right) - 3}$$
$$= \frac{\frac{21x + 28 + 20x - 28}{5x - 7}}{\frac{15x + 20 - 15x + 21}{5x - 7}}$$
$$= \frac{41x}{41}$$
$$= x$$

04) if $f(x) = \dfrac{3x + 2}{4x - 1}$ and $g(x) = \dfrac{x + 2}{4x - 3}$, Show that : $fog(x) = gof(x) = x$

SOLUTION :

$$fog(x) = f(g(x))$$
$$= \frac{3g(x) + 2}{4g(x) - 1}$$
$$= \frac{3\left(\frac{x + 2}{4x - 3}\right) + 2}{4\left(\frac{x + 2}{4x - 3}\right) - 1}$$
$$= \frac{\frac{3x + 6 + 8x - 6}{4x - 3}}{\frac{4x + 8 - 4x + 3}{4x - 3}}$$
$$= \frac{11x}{11}$$
$$= x$$

$$gof(x) = g(f(x))$$
$$= \frac{f(x) + 2}{4f(x) - 3}$$
$$= \frac{\left(\frac{3x + 2}{4x - 1}\right) + 2}{4\left(\frac{3x + 2}{4x - 1}\right) - 3}$$
$$= \frac{\frac{3x + 2 + 8x - 2}{4x - 1}}{\frac{12x + 8 - 12x + 3}{4x - 1}}$$
$$= \frac{11x}{11}$$
$$= x$$

05) if $f(x) = \dfrac{x+3}{4x-5}$ and $g(x) = \dfrac{3+5x}{4x-1}$, Show that : $fog(x) = gof(x) = x$

SOLUTION :

$$fog(x) = f(g(x))$$
$$= \frac{g(x) + 3}{4g(x) - 5}$$
$$= \frac{\left(\frac{3+5x}{4x-1}\right) + 3}{4\left(\frac{3+5x}{4x-1}\right) - 5}$$
$$= \frac{\frac{3 + 5x + 12x - 3}{4x-1}}{\frac{12 + 20x - 20x + 5}{5x-3}}$$
$$= \frac{17x}{17}$$
$$= x$$

$$gof(x) = g(f(x))$$
$$= \frac{3 + 5(f(x)}{4f(x) - 1}$$
$$= \frac{3 + 5\left(\frac{x+3}{4x-5}\right)}{4\left(\frac{x+3}{4x-5}\right) - 1}$$
$$= \frac{\frac{12x - 15 + 5x + 15}{4x-5}}{\frac{4x + 12 - 4x + 5}{4x-5}}$$
$$= \frac{17x}{17}$$
$$= x$$

SOLUTION TO Q - SET 5

01) $f(x) = 5x - 3 \; ; \; -5 \le x \le 1$

SOLUTION :

$$-5 \le x \le 1$$
$$-25 \le 5x \le 5$$
$$-25 - 3 \le 5x - 3 \le 5 - 3$$
$$-28 \le f(x) \le 2$$

Range of f is $[-28 , 2]$

02) $f(x) = 2x + 6 \; ; \; -1 \le x \le 5$

SOLUTION :

$$-1 \le x \le 5$$
$$-2 \le 2x \le 10$$
$$-2 + 6 \le 2x + 6 \le 10 + 6$$
$$4 \le f(x) \le 16$$

Range of f is $[4 , 16]$

03) $f(x) = 3 - 4x \; ; \; -4 \le x \le 2$

SOLUTION :

$$-4 \le x \le 2$$
$$16 \ge -4x \ge -8$$
$$16 + 3 \ge 3 - 4x \ge -8 + 3$$
$$19 \ge f(x) \ge -5$$

Range of f is $[-5 , 19]$

04) $f(x) = -2 - 7x \; ; \; -2 \le x \le 4$

SOLUTION :

$$-2 \le x \le 4$$
$$14 \ge -7x \ge -28$$
$$14 - 2 \ge -2 - 7x \ge -28 - 2$$
$$12 \ge f(x) \ge -30$$

Range of f is $[-30 , 12]$

05) $f(x) = 3x^2 + 5$; $-3 \leq x \leq 4$

SOLUTION :

$$-3 \leq x \leq 4$$
$$0 \leq x^2 \leq 16$$
$$0 \leq 3x^2 \leq 48$$
$$0 + 5 \leq 3x^2 + 5 \leq 48 + 5$$
$$5 \leq f(x) \leq 53$$

Range of f is $[5 , 53]$

06) $f(x) = 2x^2 - 4$; $1 \leq x \leq 4$

SOLUTION :

$$1 \leq x \leq 4$$
$$1 \leq x^2 \leq 16$$
$$2 \leq 2x^2 \leq 32$$
$$2 - 4 \leq 2x^2 - 4 \leq 32 - 4$$
$$-2 \leq f(x) \leq 28$$

Range of f is $[-2 ,28]$

07) $f(x) = 1 - 4x^2$; $-2 \leq x \leq 2$

SOLUTION :

$$-2 \leq x \leq 2$$
$$0 \leq x^2 \leq 4$$
$$0 \leq 4x^2 \leq 16$$
$$0 \geq -4x^2 \geq -16$$
$$0 + 1 \geq 1 - 4x^2 \geq -16 + 1$$
$$1 \geq f(x) \geq -15$$

Range of f is $[-15 , 1]$

08) $f(x) = 7 - 2x^2$; $2 \leq x \leq 4$

SOLUTION :

$$2 \leq x \leq 4$$
$$4 \leq x^2 \leq 16$$
$$8 \leq 2x^2 \leq 32$$
$$-8 \geq -2x^2 \geq -32$$
$$7 - 8 \geq 7 - 2x^2 \geq 7 - 32$$
$$-1 \geq f(x) \geq -25$$

Range of f is $[-25 ,-1]$

09) $f(x) = 2 - 5x^2$; $-1 \leq x \leq 3$

SOLUTION :

$$-1 \leq x \leq 3$$
$$0 \leq x^2 \leq 9$$
$$0 \leq 5x^2 \leq 45$$
$$0 \geq -5x^2 \geq -45$$
$$0 + 2 \geq 2 - 5x^2 \geq -45 + 2$$
$$2 \geq f(x) \geq -43$$

Range of f is $[-43 , 2]$

10) $f(x) = 9 - 2x^2$; $-5 \leq x \leq 3$

SOLUTION :

$$-5 \leq x \leq 3$$
$$0 \leq x^2 \leq 25$$
$$0 \leq 2x^2 \leq 50$$
$$0 \geq -2x^2 \geq -50$$
$$0 + 9 \geq 9 - 2x^2 \geq 9 - 50$$
$$9 \geq f(x) \geq -41$$

Range of f is $[-41 , 9]$

11) $f(x) = x^2 + 4x + 5 \ , x \in R$

SOLUTION :

$f(x) = x^2 + 4x + 5$

$= x^2 + 4x + 4 + 1$

$= (x + 2)^2 + 1$

Now ; $(x + 2)^2 \geq 0$

$(x + 2)^2 + 1 \geq 1$

$f(x) \geq 1$ Range of f is $[1 , \infty)$

12) $f(x) = x^2 - 8x + 10 \ , x \in R$

SOLUTION :

$f(x) = x^2 - 8x + 10$

$= x^2 - 8x + 16 + 10 - 16$

$= (x - 4)^2 - 6$

Now ; $(x - 4)^2 \geq 0$

$(x - 4)^2 - 6 \geq -6$

$f(x) \geq -6$ Range of f is $[-6 , \infty)$

13) $f(x) = x^2 - 6x + 11 \ , x \in R$

SOLUTION :

$f(x) = x^2 - 6x + 11$

$= x^2 - 6x + 9 + 11 - 9$

$= (x - 3)^2 + 2$

Now ; $(x - 3)^2 \geq 0$

$(x - 3)^2 + 2 \geq 2$

$f(x) \geq 2$ Range of f is $[2 , \infty)$

Q SET - 6 : Find the inverse of the function

01) $f(x) = 2x + 5$

$y = 2x + 5$

$y - 5 = 2x$

$x = \frac{1}{2}(y - 5)$

$\therefore f^{-1}(x) = \frac{1}{2}(x - 5)$

02) $f(x) = \frac{2x}{3} + 5$

$y = \frac{2x}{3} + 5$

$y - 5 = \frac{2x}{3}$

$x = \frac{3}{2}(y - 5)$

$f^{-1}(x) = \frac{3}{2}(x - 5)$

03) $f(x) = \frac{3x}{4} - 7$

$y = \frac{3x}{4} - 7$

$y + 7 = \frac{3x}{4}$

$x = \frac{4}{3}(y - 7)$

$f^{-1}(x) = \frac{4}{3}(x - 7)$

04) $f(x) = 3x - 4$. Find $f^{-1}(x)$. Also find $f^{-1}(9)$ and $f^{-1}(-2)$

$y = 3x - 4$

$y + 4 = 3x$

$x = \frac{1}{3}(y + 4)$

$f^{-1}(x) = \frac{1}{3}(x + 4)$

$f^{-1}(9) = \frac{1}{3}(9 + 4) = \frac{13}{3}$ & $f^{-1}(-2) = \frac{1}{3}(-2 + 4) = \frac{2}{3}$